

Editorial

# An Overview of Topological Groups: Yesterday, Today, Tomorrow

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Academic Editor: Humberto Bustince

Received: 18 April 2016; Accepted: 20 April 2016; Published: 5 May 2016

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It was in 1969 that I began my graduate studies on topological group theory and I often dived into one of the following five books. My favourite book “Abstract Harmonic Analysis” [1] by Ed Hewitt and Ken Ross contains both a proof of the Pontryagin-van Kampen Duality Theorem for locally compact abelian groups and the structure theory of locally compact abelian groups. Walter Rudin’s book “Fourier Analysis on Groups” [2] includes an elegant proof of the Pontryagin-van Kampen Duality Theorem. Much gentler than these is “Introduction to Topological Groups” [3] by Taqdir Husain which has an introduction to topological group theory, Haar measure, the Peter-Weyl Theorem and Duality Theory.

Of course the book “Topological Groups” [4] by Lev Semyonovich Pontryagin himself was a tour de force for its time. P. S. Aleksandrov, V.G. Boltyanskii, R.V. Gamkrelidze and E.F. Mishchenko described this book in glowing terms: “This book belongs to that rare category of mathematical works that can truly be called classical - books which retain their significance for decades and exert a formative influence on the scientific outlook of whole generations of mathematicians”.

The final book I mention from my graduate studies days is “Topological Transformation Groups” [5] by Deane Montgomery and Leo Zippin which contains a solution of Hilbert’s fifth problem as well as a structure theory for locally compact non-abelian groups. These five books gave me a good feeling for the most significant research on locally compact group theory in the first 60 years of the twentieth century. My own contribution to understanding the structure of locally compact abelian groups was a small book “Pontryagin Duality and the Structure of Locally Compact Abelian Groups” [6] which was translated into Russian and served to introduce a generation of young Soviet mathematicians to this topic.

Far from locally compact groups, A.A. Markov [7,8] introduced the study of free topological groups. This was followed up by M.I. Graev in 1948 [9] with a slightly more general concept. Free topological groups are an analogue of free groups in abstract group theory. Markov gave a very long construction of the free topological group on a Tychonoff space and also proved its uniqueness. Graev’s proof is also long. Shorter proofs appeared after a few years. Today one derives the existence of Markov and Graev free topological groups from the Adjoint Functor Theorem. Free topological groups have been an active area of research to this day, especially by Alexander Vladimirovich Arhangel’skii of Moscow State University and his former doctoral students and they have produced a wealth of deep and interesting results.

Now let me turn to this volume. My aim for “Topological Groups: Yesterday, Today, Tomorrow” is for these articles to describe significant topics in topological group theory in the 20th century and the early 21st century as well as providing some guidance to the future directions topological group theory might take by including some interesting open questions.

“In 1900 David Hilbert presented a seminal address to the International Congress of Mathematicians in Paris. In this address, he initiated a program by formulating 23 problems,

which influenced a vast amount of research of the 20th century. The fifth of these problems asked whether every locally-Euclidean topological group admits a Lie group structure. This motivated an enormous volume of work on locally-compact groups during the first half of the 20th century. It culminated in the work of Gleason, Iwasawa, Montgomery, Yamabe and Zippin, yielding a positive answer to Hilbert's fifth problem and exposing the structure of almost connected locally-compact groups [5]. (Recall that a topological group  $G$  is called almost connected [10] if the quotient group  $G/G_0$ , modulo the connected component  $G_0$  of the identity, is compact. The class of almost connected groups includes all compact groups and all connected locally-compact groups.). The advances in the second half of the 20th century shed much light on the structure and representation theory of locally compact groups" is how Karl Heinrich Hofmann and Sidney A. Morris began their article *Pro-Lie Groups: A Survey with Open Problems* in this volume.

While the class of locally compact abelian groups has the beautiful Pontryagin-van Kampen Duality from which the structure of locally compact abelian groups can be described (see [6]), the structure theory of compact groups has not been derived from any of the various Duality Theorems for compact groups. This led Hofmann and Morris to establish and use a Lie Theory for compact groups to provide a complete description of the structure of compact groups in [11]. They then used in [10] the same Lie Theory approach to establish the structure theory of (almost) connected locally compact groups. As the class of locally compact groups is not closed even under infinite products, they introduced the class of pro-Lie Groups which is a natural extension of the classes of finite-dimensional Lie groups, locally compact abelian groups, compact groups and connected locally compact groups and used the Lie Theory to describe completely the structure of almost connected pro-Lie groups. Their article *Pro-Lie Groups: A Survey with Open Problems* provides an up-to-date summary of pro-Lie groups and lists 12 interesting questions. Probably the most interesting of these is

**Question 2.** Let  $G$  be a pro-Lie group with identity component  $G_0$ . Is  $G/G_0$  complete (and therefore, prodiscrete)?

Over the last 50 years there has been a steady development of the theory of pseudocompact topological groups. In their article *Non-abelian Pseudocompact Groups* in this volume Wis Comfort and Dieter Remus survey the historical development of the theory of pseudocompact topological groups. They report that "Many of the results we cite, especially the older results, require an abelian hypothesis; some questions, definitions and results make sense and are correct without that hypothesis, however, and we emphasize these. Thus, this paper has two goals: (1) to provide an overview of the (by now substantial) literature on pseudocompact groups; and (2) to offer several new results about non-abelian pseudocompact groups."

In particular Comfort and Remus examine "three recently-established theorems from the literature:

- (A) (2006) Every non-metrizable compact abelian group  $K$  has  $2^{|K|}$ -many proper dense pseudocompact subgroups.
- (B) (2003) Every non-metrizable compact abelian group  $K$  admits  $2^{2^{|K|}}$ -many strictly finer pseudocompact topological group refinements.
- (C) (2007) Every non-metrizable pseudocompact abelian group has a proper dense pseudocompact subgroup and a strictly finer pseudocompact topological group refinement.

(Theorems (A), (B) and (C) become false if the non-metrizable hypothesis is omitted.)". The authors ask: What happens to (A), (B), (C) and to similar known facts about pseudocompact abelian groups if the abelian hypothesis is omitted? Are the resulting statements true, false, true under certain natural additional hypotheses, etc.? Several new results responding in part to these questions are given, and several specific additional questions are posed. One conjecture they mention is due to Comfort and van Mill.

**Conjecture 5.4.1.** Let  $G$  be an abelian group which admits a pseudocompact group topology. Then the supremum of the pseudocompact group topologies on  $G$  coincides with the largest totally bounded group topology on  $G$  (that is, the topology induced on  $G$  by  $\text{Hom}(G, \mathbb{T})$ ).

We mention two of the questions they ask:

**Problem 5.7.2.** Does every infinite compact group  $K$  have  $2^{|K|}$ -many non-measurable subgroups (of cardinality  $|K|$ )?

**Problem 8.2.11.** \*Let  $(K, \mathcal{T})$  be a profinite group of uncountable weight.

(a) Does  $\mathcal{T}$  admit a proper pseudocompact refinement of maximal weight  $2^{|K|}$ ?

(b) Are there  $2^{2^{|K|}}$ -many pseudocompact group topologies on  $K$  which are finer than  $\mathcal{T}$ ?

The next paper we discuss here is *Free Boolean Topological Groups* by Ol'ga Sipacheva. She introduces her paper as follows: "In the very early 1940s, A. A. Markov [7,8] introduced the free topological group  $F(X)$  and the free Abelian topological group  $A(X)$  on an arbitrary completely regular Hausdorff topological space  $X$  as a topological-algebraic counterpart of the abstract free and free Abelian groups on a set; he also proved the existence and uniqueness of these groups. During the next decade, Graev [9,12], Nakayama [13], and Kakutani [14] simplified the proofs of the main statements of Markov's theory of free topological groups, generalized Markov's construction, and proved a number of important theorems on free topological groups. In particular, Graev generalized the notions of the free and the free Abelian topological group on a space  $X$  by identifying the identity element of the free group with an (arbitrary) point of  $X$  (the free topological group on  $X$  in the sense of Markov coincides with Graev's group on  $X$  plus an isolated point), described the topology of free topological groups on compact spaces, and extended any continuous pseudometric on  $X$  to a continuous invariant pseudometric on  $F(X)$  (and on  $A(X)$ ) which is maximal among all such extensions [9].

This study stimulated Mal'tsev, who believed that the most appropriate place of the theory of abstract free groups was in the framework of the general theory of algebraic systems, to introduce general free topological algebraic systems. In 1957, he published the large paper [15], where the basics of the theory of free topological universal algebras were presented.

Yet another decade later, Morris initiated the study of free topological groups in the most general aspect. Namely, he introduced the notion of a variety of topological groups (A definition of a variety of topological groups (determined by a so-called varietal free topological group) was also proposed in 1951 by Higman [16]; however, it is Morris' definition which has proved viable and developed into a rich theory.) and a full variety of topological groups and studied free objects of these varieties [17–19] (see also [20]). Varieties of topological groups and their free objects were also considered by Porst [21], Comfort and van Mill [22], Kopperman, Mislove, Morris, Nickolas, Pestov, and Svetlichny [23], and other authors. Special mention should be made of Dikranjan and Tkachenko's detailed study of varieties of Abelian topological groups with properties related to compactness [24].

The varieties of topological groups in which free objects have been studied best are, naturally, the varieties of general and Abelian topological groups; free and free Abelian precompact groups have also been considered (see, e.g., [25]). However, there is yet another natural variety—Boolean topological groups. Free objects in this variety and its subvarieties have been investigated much less extensively, although they arise fairly often in various studies (especially in the set-theoretic context). The author is aware of only two published papers considering free Boolean topological groups from a general point of view: [26], where the topology of the free Boolean topological group on a compact metric space was explicitly described, and [27], where the free Boolean topological groups on compact initial segments of ordinals were classified (see also [28]). The purpose of this paper is to draw attention to these very interesting groups and give a general impression of them. We collect some (known and new) results on free Boolean topological groups, which describe both properties which these groups share with free or free Abelian topological groups and properties specific of free Boolean groups.

We mention here Theorem 8: If  $\dim X = 0$ , then  $\text{ind}B(X) = 0$ , which can be proved much more easily than the analogous result for free topological groups. By contrast, Proposition 9 says: The free Abelian topological group on any connected space has infinitely many connected components, however the free Boolean topological group on any connected space has two connected components. We record here a few of Sipacheva's questions:

**Problem 3.** Does there exist a space  $X$  such that  $B(X)$  is normal, but  $X^2$  is not?

**Problem 4.** Describe spaces  $X$  for which  $B(X)$  is Lindelöf. Does there exist a space  $X$  such that  $B(X)$  is Lindelöf, but  $X$  is not?

**Problem 5.** Does there exist a space  $X$  for which  $B(X)$  is normal (Lindelöf, ccc), but  $F(X)$  or  $A(X)$  is not?

**Problem 6.** Is it true that  $B(X)$  is Weil complete for any Dieudonné complete space  $X$ ?

**Problem 7.** Is it true that the free (free Boolean) topological group of any stratifiable space is stratifiable?

The article *On  $T$ -Characterized Subgroups of Compact Abelian Groups* by Saak Gabrielyan addresses  $T$ -sequences in compact abelian groups. A sequence  $\{u_n\}$  in an Abelian group  $G$  is called a  $T$ -sequence if there is a Hausdorff group topology on  $G$  relative to which  $\lim_n u_n = 0$ . A subgroup  $H$  of an infinite compact Abelian group  $X$  is said to be  $T$ -characterized if there is a  $T$ -sequence  $\mathbf{u} = \{u_n\}$  in the dual group of  $X$  such that  $H = \{x \in X : (u_n, x) \rightarrow 1\}$ . The author summarizes the results in this paper as follows: "We show that a closed subgroup  $H$  of  $X$  is  $T$ -characterized if and only if  $H$  is a  $G_\delta$ -subgroup of  $X$  and the annihilator of  $H$  admits a Hausdorff minimally almost periodic group topology. All closed subgroups of an infinite compact Abelian group  $X$  are  $T$ -characterized if and only if  $X$  is metrizable and connected. We prove that every compact Abelian group  $X$  of infinite exponent has a  $T$ -characterized subgroup which is not an  $F_\sigma$ -subgroup of  $X$ , that gives a negative answer to Problem 3.3 in [29]".

The next paper we introduce is *Characterized Subgroups of Topological Abelian Groups* by Dikran Dikranjan, Anna Giordano Bruno and Danele Impieri. Historically, characterized subgroups were studied exclusively in the case of the circle group  $\mathbb{T}$  in the context of Diophantine approximation, dynamical systems and ergodic theory, see for example [30]. A subgroup  $H$  of an abelian topological group  $X$  is said to be characterized by a sequence  $\mathbf{v} = (v_n)$  of characters of  $X$  if  $H = \{x \in X : v_n(x) \rightarrow 0 \text{ in } \mathbb{T}\}$ . The authors say "we introduce the relevant class of auto-characterized groups (namely, the groups that are characterized subgroups of themselves by means of a sequence of non-null characters); in the case of locally compact abelian groups, these are proven to be exactly the non-compact ones. As a by-product of our results, we find a complete description of the characterized subgroups of discrete abelian groups". Amongst the questions presented in the paper, we mention:

**Question 5.** Are the closed  $G_\delta$ -subgroups of a precompact abelian always  $N$ -characterized? (This is equivalent to asking if there exists a continuous injection from  $X/F$  into  $\mathbb{T}^n$  for every closed  $G_\delta$ -subgroup  $F$  of a precompact abelian group  $X$ .)

In the paper *Fixed Points of Local Actions of Lie Groups on Real and Complex 2-Manifolds*, Morris W. Hirsch surveys "old and new results on fixed points of local actions by Lie groups  $G$  on real and complex 2-manifolds. The theme is to find conditions guaranteeing that a compact set of fixed points of a 1-parameter subgroup contains a fixed point of  $G$ ." The classical results of Poincaré (1885) [31], Hopf (1925) [32] and Lefschetz (1937) [33] yield.

**Theorem.** Every flow on a compact manifold of nonzero Euler characteristic has a fixed point.

The earliest papers I've found on fixed points for actions of other nondiscrete Lie group are those of P. A. Smith [34] (1942) and H. Wang [35], (1952). Then came Borel [36] with

**Theorem.** If  $H$  is a solvable, irreducible algebraic group over an algebraically closed field  $\mathbb{K}$ , every algebraic action of  $H$  on a complete algebraic variety over  $\mathbb{K}$  has a fixed point."

In this paper Hirsch, in particular, puts into context the results of Sommesse (1973) [37], Lima (1964) [38], Plante (1986) [39], Bonatti (1992) [40], Hirsch (2001) [41], Hirsch (2010) [42], Hirsch (2013) [43] and Hirsch (2014) [44].

Next we turn to the survey paper *Open and Dense Topological Transitivity of Extensions by Non-compact Fiber of Hyperbolic Systems – a Review* by Viorel Nitica and Andrei Török. They summarize their paper as follows: “Currently there is great renewed interest in proving topological transitivity of various classes of continuous dynamical systems. Even though this is one of the most basic dynamical properties that can be investigated, the tools used by various authors are quite diverse and are strongly related to the class of dynamical systems under consideration. The goal of this survey article is to present the state of art for the class of Hölder extensions of hyperbolic systems with non-compact connected Lie group fiber. The hyperbolic systems we consider are mostly discrete time. In particular, we address the stability and genericity of topological transitivity in large classes of such transformations. The paper lists several open problems, conjectures and tries to place this topic of research in the general context of hyperbolic and topological dynamics”. The Main Conjecture is:

**Conjecture 6.** Assume that  $X$  is a hyperbolic basic set for  $f : X \rightarrow X$  and  $\Gamma$  is a finite-dimensional connected Lie group. Among the Hölder cocycles  $\beta : X \rightarrow X$  with subexponential growth that are not cohomologous to a cocycle with values in a maximal subsemigroup of  $\Gamma$  with non-empty interior, there is a Hölder open and dense set for which the extension  $f_\beta$  is transitive.

The conjecture is proved for various classes of Lie groups. The techniques used so far are quite diverse and seem to depend heavily on the particular properties of the group that appears in the fiber.

The next paper we discuss is *Locally Quasi-Convex Compatible Topologies on a Topological group* by Lydia Außenhofer, Dikran Dikranjan and Elena Martín-Peinador.

“Varopoulos posed the question of the description of the group topologies on an abelian group  $G$  having a given character group  $H$ , and called them compatible topologies for the duality  $(G; H)$ , [45]. As the author explains, the question is motivated by Mackey’s Theorem, which holds in the framework of locally convex spaces. He treated the question within the class of locally precompact abelian groups. Later on, this problem was set in a bigger generality in [46]; namely, within the class of locally quasi-convex groups. This is a class of abelian topological groups which properly contains the class of locally convex spaces, a fact which makes the attempt to generalize the Mackey-Arens Theorem more natural”.

The authors summarize their results as follows: “For a locally quasi-convex topological abelian group  $(G, \tau)$  we study the poset  $\mathcal{C}(G, \tau)$  of all locally quasi-convex topologies on  $G$  that are compatible with  $\tau$  (i.e., have the same dual as  $(G, \tau)$  ordered by inclusion. Obviously, this poset has always a bottom element, namely the weak topology  $\sigma(G, \hat{G})$ . Whether it has also a top element is an open question. We study both quantitative aspects of this poset (its size) and its qualitative aspects, e.g., its chains and anti-chains. Since we are mostly interested in estimates ‘from below’, our strategy consists in finding appropriate subgroups  $H$  of  $G$  that are easier to handle and show that  $\mathcal{C}(H)$  and  $\mathcal{C}(G/H)$  are large and embed, as a poset, in  $\mathcal{C}(G, \tau)$ . Important special results are: (i) If  $K$  is a compact subgroup of a locally quasi-convex group  $G$ , then  $\mathcal{C}(G)$  and  $\mathcal{C}(G/K)$  are quasi-isomorphic; (ii) If  $D$  is a discrete abelian group of infinite rank, then  $\mathcal{C}(D)$  is quasi-isomorphic to the poset  $\mathfrak{F}_D$  of filters on  $D$ . Combining both results, we prove that for a LCA (locally compact abelian) group  $G$  with an open subgroup of infinite co-rank (this class includes, among others, all non  $\sigma$ -compact LCA groups), the poset  $\mathcal{C}(G)$  is as big as the underlying topological structure of  $(G, \tau)$  (and set theory) allow. For a metrizable connected compact group  $X$  the group of null-sequences  $G = c_0(X)$  with the topology of uniform convergence is studied. We prove that  $\mathcal{C}(G)$  is quasi-isomorphic to  $\mathcal{P}(\mathbb{R})$ .” Three questions are recorded below:

**Question 7.3.** Let  $G$  be a non-precompact second countable Mackey group. Is it true that  $|\mathcal{C}(G)| \geq \mathfrak{c}$ .

**Problem 7.4.** Find sufficient conditions for a metrizable precompact group  $G$  to be Mackey (i.e., have  $|\mathcal{C}(G)| = 1$ .)

**Conjecture 7.6.** [Mackey dichotomy] For a locally compact group  $G$ , one has either  $|\mathcal{C}(G)| = 1$  or  $|\mathcal{C}(G)| \geq \mathfrak{c}$ .

Last, but certainly not least, we mention *Lindelöf  $\Sigma$ -Spaces and  $\mathbb{R}$ -Factorizable Paratopological Groups* by Mikhail Tkachenko. He summarizes the results as follows: “We prove that if a paratopological group  $G$  is a continuous image of an arbitrary product of regular Lindelöf  $\Sigma$ -spaces, then it is  $\mathbb{R}$ -factorizable and has countable cellularity. If in addition  $G$  is regular, then it is totally  $\omega$ -narrow, and satisfies  $cel_\omega(G) \leq \omega$ , and the Hewitt-Nachbin completion of  $G$  is again an  $\mathbb{R}$ -factorizable paratopological group”. A curious consequence of the above is Corollary 14: The Sorgenfrey line is not a continuous image of any product of regular Lindelöf  $\Sigma$ -spaces. We conclude by mentioning three questions in this paper:

**Problem 15.** Let a (Hausdorff) paratopological group  $G$  be a continuous image of a product of a family of Lindelöf  $\Sigma$ -spaces. Does  $G$  have the Knaster property? Is it  $\omega$ -narrow?

**Problem 17.** Let a Hausdorff (regular) paratopological group  $G$  be a continuous image of a dense subspace of a product of separable metrizable spaces. Is  $G$  perfectly  $\kappa$ -normal or  $\mathbb{R}$ -factorizable?

**Problem 18.** Does every upper quasi-uniformly continuous quasi-pseudometric on an arbitrary product of Lindelöf  $\Sigma$ -spaces depend at most on countably many coordinates?

In conclusion, the collection of articles in this volume should give the reader an overview of topological group theory as it developed over the last 115 years, as well as the richness of current research. In this Editorial I have listed some of the open questions in these papers which interested me, but the papers themselves contain many more. My hope is that you, the reader, will solve some of these problems and contribute to the future development of topological group theory.

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