

A topological generalization of the Higman–Neumann–Neumann theorem

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Abstract. We generalize in a substantial way the celebrated result by Graham Higman, Bernhard Neumann and Hanna Neumann on embedding countable groups into 2-generator groups as follows: every countable topological group is isomorphic to a topological subgroup of a topological group algebraically generated by two elements. A number of corollaries are derived. In particular, we characterize those topological groups embeddable into groups with two topological generators: they are the groups covered by countably many translations of each neighbourhood of the identity and having weight at most c . In particular, they include all separable topological groups.

0 Introduction

A famous result of Graham Higman, Bernhard Neumann and Hanna Neumann [9] asserts that every countable group is isomorphic to a subgroup of a 2-generator group. Their proof uses free products with amalgamations. Later Bernhard and Hanna Neumann gave a proof using wreath products [14]. Subsequently the latter proof was converted into a very transparent form, and there is an excellent exposition by Fred Galvin in the American Mathematical Monthly [5]. (See [6] for even more refined results using the same construction.)

Here we show that the proof as presented by Galvin can be reconstructed at the level of topological dynamics so as to lead to the following general result for countable topological groups.

Theorem A. *Let G be any countable topological group. Then there exists a topological group H which has G as a topological subgroup and which is algebraically generated by some $a, b \in H$.*

The original Higman–Neumann–Neumann theorem is recovered in the special case where the original group G has the discrete topology.

As a direct corollary of Theorem A, one deduces the following new result: every